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Sequence

[2]

Question 1

Here are the first four terms of a sequence.

23 17 11 5

(-6) (-6)

(a) Find the next term. [1]

-1

(b) Find the *n*th term. [2]

$n^{th} = 23 + (n-1)(-6)$

$= 23 - 6n + 6 = 29 - 6n$

Question 2

7, 5, 3, 1, – 1, …

(-2) (-2)

(a) Find the next term in this sequence. [1]

-3

(b) Find the *n*th term of the sequence. [2]

$n^{th} = 7 + (n-1)(-2)$

$= 7 - 2n + 2 = 9 - 2n$

Question 3

Find the *n*th term of each sequence.

(a) 4, 8, 12, 16, 20, ... [1]

+4

$n^{th} = 4 + (n-1)4$

$= 4n$

(b) 11, 20, 35, 56, 83, ... [2]

a+b+c

+9 +15 +21

3a+b

+6 +6

2a

$2a = 6$ $a = 3$

$3a + b = 9$

$9 + b = 9$

$b = 0$

$a+b+c = 11$

$c = 11 - 3$

$c = 8$

n^{th} term $= 3n^2 + 8$

Question 4

2a +4 +4

3a+b +6 +10 +14

a+b+c 5, 11, 21, 35, 53, ...

Find the *n*th term of this sequence. [2]

$2a = 4$ $a = 2$

$3a + b = 6$

$6 + b = 6$

$b = 0$

$a+b+c = 5$

$2 + c = 5$

$c = 3$

$n^{th} = 2n^2 + 3$

Question 5

These are the first five terms of a sequence.

13 8 3 –2 –7

Find the *n*th term of this sequence. [2]

$n^{th} = 13 + (n-1)(-5)$

$= 13 - 5n + 5$

$= 18 - 5n$

Question 6

32 25 18 11 4

These are the first 5 terms of a sequence.

Find

(a) the 6th term, [1]

-3

(b) the *n*th term, [2]

$n^{th} = 32 + (n-1)(-7)$

$= 32 - 7n + 7 = 39 - 7n$

(c) which term is equal to –332. [2]

$39 - 7n = -332$

$-7n = -371$

$n = 53$

Question 7

The first five terms of a sequence are shown below.

13 9 5 1 –3

Find the *n*th term of this sequence. [2]

$n^{th} = 13 + (n-1) - 4$

$= 13 - 4n + 4$

$= 17 - 4n$

Question 8

A sequence is given by $u_1 = \sqrt{1}$, $u_2 = \sqrt{3}$, $u_3 = \sqrt{5}$, $u_4 = \sqrt{7}$, …

(a) Find a formula for u_n, the *n*th term. [2]

$1 + (n-1)(2)$

$1 + 2n - 2 = 2n - 1$

$n^{th} = \sqrt{2n-1}$

(b) Find u_{29}. [1]

$u_{29} = \sqrt{57}$

Question 10

For each of the following sequences, write down the next term.

(a) 2, 3, 5, 8, 13, … [1]

21

(b) x, $6x$, $30x$, $120x$, … [1]

×6 (6), ×5 (5), ×4 (4), ×3 (3)

$360x^2$

[1]

(c) 2, 6, 18, 54, 162, ...

×3, ×3

486

Question 11

For the sequence $5\frac{1}{2}$, 7, $8\frac{1}{2}$, 10, $11\frac{1}{2}$, …

1.5, 1.5, 1.5

(a) find an expression for the nth term, [2]

$n^{th} = 5.5 + (n-1)1.5$

$= 5.5 + 1.5n - 1.5$

$= 4 + 1.5n$

(b) work out the 100th term. [1]

$100^{th} = 4 + 150$

$= 154$

Question 12

Write down the next term in each of the following sequences.

(a) 8.2, 6.2, 4.2, 2.2, 0.2, … [1]

−2, −2

−1.8

(b) 1, 3, 6, 10, 15, … [1]

+2, +3, +4, +5

21

Question 14

8, 15, 22, 29, 36, ……..

A sequence of numbers is shown above.

(a) Find the 10th term of the sequence. [1]

71

(b) Find the nth term of the sequence. [1]

$n^{th} = 8 + (n-1)7$

$= 8 + 7n - 7$

$= 1 + 7n$

(c) Which term of the sequence is equal to 260? [1]

$1 + 7n = 260$

$7n = 259$

$n = 37$

Question 15

The first five terms of a sequence are 4, 9, 16, 25, 36, …
Find

(a) the 10th term, [1]

121

(b) the nth term. [1]

$n^{th} = (n+1)^2$

Question 1

Find the nth term of each sequence.

(a) 7, 13, 19, 25, 31, … [2]

+6 +6

$7+(n-1)6 = 7+6n-6$

$= 6n+1$

(b) 9, 16, 25, 36, 49, … [2]

$(n+2)^2$

Question 2

Find the nth term of each of these sequences.

(a) 16, 19, 22, 25, 28, … [2]

+3 +3

$16+(n-1)3 = 16+3n-3 = 13+3n$

(b) 1, 3, 9, 27, 81, … [2]

3^{n-1}

Question 3

The nth term of a sequence is $an^2 + bn$.

(a) Write down an expression, in terms of a and b, for the 3rd term. [1]

$9a+3b$

(b) The 3rd term of this sequence is 21 and the 6th term is 96.

Find the value of a and the value of b.
You must show all your working. [4]

$9a+3b=21$ ×2

$36a+6b=96$

$-\ 18a+6b=42$

$18a = 54$

$a = 3$

$27+3b=21$

$3b=-6$

$b=-2$

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Question 5

Find the nth term in each of the following sequences.

(a) $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots\ldots$ [1]

$\frac{n}{n+2}$

(b) 0, 3, 8, 15, 24, ... [2]

3 5 7 9

2 2 2

$2a = 2$
$a = 1$

$3a + b = 3$
$3 + b = 3$
$b = 0$

$a + b + c = 0$
$1 + c = 0$
$c = -1$

$n^{th} = n^2 - 1$

Question 7

Pattern 1 Pattern 2 Pattern 3

The first three patterns in a sequence are shown above.

(a) Complete the table.

Pattern number	1	2	3	4
Number of dots	5	8	11	14

3 3

[1]

(b) Find a formula for the number of dots, d, in the nth pattern. [1]

$n^{th} = 5 + (n-1)3$

$= 5 + 3n - 3 = 3n + 2$

(c) Find the number of dots in the 60th pattern. [1]

$180 + 2 = 182$

(d) Find the number of the pattern that has 89 dots. [1]

$3n + 2 = 89$
$3n = 87$
$n = 29$

Question 2

(a) Complete the table for the four sequences A, B, C and D.

	Sequence	Next term	*n*th term
A	2 5 8 11	14	3n-1
B	20 -6 14 -6 8 -6 2	-4	26-6n
C	1 4 9 16	25	n^2
D	0 2 6 12	20	n^2-n

)-1)-2)-3)-4

[10]

2+(n-1)3
2+3n-3 = 3n-1

20+(n-1)-6
20-6n+6
26-6n

(b) The sum of the first n terms of a sequence is $\frac{n(3n+1)}{2}$.

(i) When the sum of the first n terms is 155, show that $3n^2 + n - 310 = 0$. [2]

$\frac{n(3n+1)}{2} = 155$

$3n^2+n = 310$

$3n^2+n-310=0$

(ii) Solve $3n^2 + n - 310 = 0$. [3]

3n + 31 +31
n - 10 -30

(3n+31)(n-10) = 0

$n = -\frac{31}{3}$ or $n = 10$

(reject)

(iii) Complete the statement.

The sum of the first ..10.. terms of this sequence is 155. [1]

Question 3

The first three diagrams in a sequence are shown below.
The diagrams are made by drawing lines of length 1 cm.

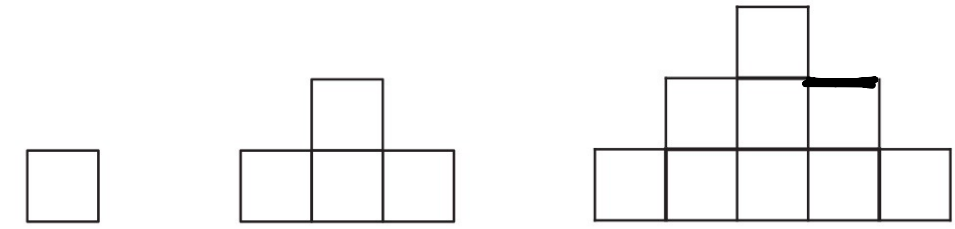

Diagram 1 Diagram 2 Diagram 3

(a) The areas of each of the first three diagrams are shown in this table.

Diagram	1	2	3
Area (cm^2)	1	4	9

(i) Find the area of Diagram 4. [1]

16

(ii) Find, in terms of n, the area of Diagram n. [1]

n^2

(b) The numbers of 1 cm lines needed to draw each of the first three diagrams are shown in this table.

Diagram	1	2	3	43
Number of 1 cm lines	4	13	26	

9 13 17 4

(i) Find the number of 1 cm lines needed to draw Diagram 4. [1]

(ii) In which diagram are 118 lines of length 1 cm needed? [1]

$2a = 4$ $3a + b = 9$ $a + b + c = 4$

$a = 2$ $6 + b = 9$ $c = -1$

$b = 3$

$2n^2 + 3n - 1 = 118$

$2n^2 + 3n - 119 = 0$

$(n - 7)(2n + 17) = 0$

$n = 7$ or $n = -\frac{17}{2}$ (reject)

(c) The **total** number of 1 cm lines needed to draw both Diagram 1 and Diagram 2 is 17.
The **total** number of 1 cm lines needed to draw all of the first n diagrams is

$$\frac{2}{3}n^3 + an^2 + bn.$$

Find the value of a and the value of b.
Show all your working. [6]

$\frac{16}{3} + 4a + 2b = 17$

$16 + 12a + 6b = 51$

$12a + 6b = 35$ — ①

$\frac{2}{3} + a + b = 4$

$3a + 3b = 10 \times 2$

$6a + 6b = 20$

$-(12a + 6b = 35)$

$-6a = -15$

$a = \frac{5}{2}$

$\frac{15}{2} + 3b = 10$

$3b = \frac{5}{2}$

$b = \frac{5}{6}$

Question 4

Complete the table for each sequence.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term		nth term
A	15	8 (−7)	1 (−7)	−6 (−7)	−13	−20		22 − 7n
B	$\frac{5}{18}$	$\frac{6}{19}$	$\frac{7}{20}$	$\frac{8}{21}$	$\frac{9}{22}$	$\frac{10}{23}$		$\frac{n+4}{n+17}$
C	2	5 (3)	10 (5)	17 (7)	26 (9)	36		n^2+1
D	2	6 (×3)	18 (×3)	54 (×3)	162 (×3)	486		$2 \times 3^{n-1}$

[11]

$15 + (n-1) \times -7 = 15 - 7n + 7 = 22 - 7n$

$5 + (n-1)$

$n + 4$

$n + 17$

$2a = 2$

$a = 1$

$3a + b = 3$

$b = 0$

$a + b + c = 2$

$c = 1$

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Question 2

Complete the table for the following sequences.
The first row has been completed for you.

	Sequence	Next two terms	nth term	
	1 5 9 13	17 21	$4n-3$	
(a)	12 (+9) 21 (+9) 30 (+9) 39	48 57	$3+9n$	[3]
(b)	80 (-6) 74 (-6) 68 (-6) 62	56 50	$86-6n$	[3]
(c)	1 (+1) 8 (+2) 27 (+3) 64 (+4)	125 216	n^3	[2]
(d)	2 10 30 68	130 222	n^3+n	[2]

$12+(n-1)9$ $\quad$ $80+(n-1)-6$

$12+9n-9=$ $\quad$ $80-6n+6$